

Math 2331: Linear algebra

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Tiny Instagram (arrow = follow):

$A \rightleftarrows B$

$\downarrow \quad \searrow \quad \downarrow \uparrow$

$C \longrightarrow D$

Q Who's the biggest influencer?

Idea Influence depends on followers

A 1 Q Are A + C really

B 2 equally influential?

C 1 Q Is D really the most

D 3 influential?

Idea Not all followers are equal

- Being followed by someone influential is a big deal
- Being followed by someone who follows everyone isn't

$$A = \frac{1}{2}B$$

$$B = \frac{1}{3}A + D$$

$$C = \frac{1}{3}A$$

$$D = \frac{1}{3}A + \frac{1}{2}B + C$$

$$B = 2A$$

$$C = \frac{1}{3}A$$

$$D = \frac{5}{3}A$$

Influence is all relative, so normalize by (for example) requiring that  $A=1$ :

	Country	baby PageRank
A	1	1
B	2	2
C	1	$\frac{1}{3}$
D	3	$\frac{5}{3}$

Problem Instagram has 1,000,000,000 users

Solution Get more organized

First, we encode the expression on the RHS in a square array called a matrix:

$$M = \begin{bmatrix} 0 & 1/2 & 0 & 0 \\ 1/3 & 0 & 0 & 1 \\ 1/3 & 0 & 0 & 0 \\ 1/3 & 1/2 & 1 & 0 \end{bmatrix}$$

We encode the values we wish to determine in a column vector:

$$\vec{v} = \begin{bmatrix} A \\ B \\ C \\ D \end{bmatrix}$$

The system of equations is encoded by the single matrix equation

$$\vec{v} = M\vec{v}.$$

Linear algebra gives us tools (algorithms) for solving equations like this by performing operations on the entries of  $M$  ("row reduction," next time).

Problem Instagram has 1,000,000,000 users!

Solution Don't solve the equations (get close)

Linear algebra can do this too ("discrete dynamical systems", later).

[Worksheet]

$$70 - 2P_1 + P_2 = D_1 = S_1 = -14 + 3P_1$$

$$105 + P_1 - P_2 = D_2 = S_2 = -7 + 2P_2$$

$$5P_1 - P_2 = 84$$

$$P_1 - 3P_2 = -112$$

"Solving the system" means getting from

$$\begin{bmatrix} 5P_1 - P_2 = 84 \\ P_1 - 3P_2 = -112 \end{bmatrix} \text{ to } \begin{bmatrix} P_1 = ? \\ P_2 = ? \end{bmatrix}$$

↓ swap

$$\begin{bmatrix} P_1 - 3P_2 = -112 \\ 5P_1 - P_2 = 84 \end{bmatrix} \xrightarrow{-5 \times \#1} \begin{bmatrix} P_1 - 3P_2 = -112 \\ 14P_2 = 644 \end{bmatrix}$$

↓ #2/14

$$\begin{bmatrix} P_1 = 26 \\ P_2 = 46 \end{bmatrix} \xleftarrow{+3 \times \#2} \begin{bmatrix} P_1 - 3P_2 = -112 \\ P_2 = 46 \end{bmatrix}$$