Last time

$$\begin{bmatrix} 1 & ? & 0 & 0 & ? \\ & & 1 & 0 & \vdots \\ 0 & & & 1 & \end{bmatrix}$$

- Systems of linear equations
- Coefficient + augmented matrices
- RREF
- Row operations

(I) <u>Swap</u> two rows
(II) <u>Add</u> a multiple of a row
(III) <u>Scale</u> a row

<u>Fact</u> Every matrix A has a unique RREF, which can be obtained from A by applying row operations. The two matrices represent systems of equations with the same solutions.

Row reduction algorithm ("Gauss-Jordan elimination")

① Row by row, top to bottom

a) If row has nonzero entry, scale first to be 1 (III)

b) Eliminate entries above and below (II)

② Rearrange the rows (I)

As we saw last time, a system need not have a solution. If it does, it need not be unique.

Solution behavior	matrix behavior
no solutions ("inconsistent")	RREF of augmented matrix has a pivot in the last column
unique solution	RREF of augmented matrix has a pivot in every column but the last
infinitely many solutions	RREF of augmented matrix has no pivots in both the last column and at least one other column