WORKSHEET 2/8/23
MATH 2331, SPRING 2023

You may take for granted that the reduced row-echelon form of the matrix $\begin{bmatrix} 1 & 4 & 7 \\ 2 & 5 & 8 \\ 3 & 6 & 9 \end{bmatrix}$ is the

matrix $\begin{bmatrix} 1 & 0 & -1 \\ 0 & 1 & 2 \\ 0 & 0 & 0 \end{bmatrix}$.

(1) Are the vectors $\vec{v}_1 = (1, 2, 3)$, $\vec{v}_2 = (4, 5, 6)$, and $\vec{v}_3 = (7, 8, 9)$ linearly independent?

(2) How large could a collection of linearly independent vectors in $\mathbb{R}^m$ be?

(3) Do the vectors $\vec{v}_1 = (1, 2, 3)$, $\vec{v}_2 = (4, 5, 6)$, and $\vec{v}_3 = (7, 8, 9)$ span $\mathbb{R}^3$?

(4) How small could a set of vectors spanning $\mathbb{R}^m$ be?

(5) Suppose that $\vec{w}_1$, $\vec{w}_2$, and $\vec{w}_3$ are linearly independent vectors. How many ways could there be to write another vector $\vec{w}$ as a linear combination of these vectors?