Last time

- Subspaces
- Spans
- Redundancy, relations, linear independence

columns of A linearly independent $\iff$ $A\vec{x}=\vec{0}$ has a unique solution $\iff$ $\ker A = \{\vec{0}\}$ $\iff$ $rk(A)=n$

So a collection of more than m vectors in $\mathbb{R}^m$ can't be linearly independent!

columns of A span $\mathbb{R}^m$ $\iff$ $A\vec{x}=\vec{b}$ has a solution for every $\vec{b}$ $\iff$ $\operatorname{im} A = \mathbb{R}^m$ $\iff$ $rk(A)=m$

So a collection of fewer than m vectors in $\mathbb{R}^m$ can't span!

Def A **basis** for a subspace V of $\mathbb{R}^n$ is a linearly independent collection of vectors in V that span V.

Every basis for a subspace has the same number of vectors, called the **dimension** $\dim V$ of the subspace.

Ex $\dim \mathbb{R}^m = m$

Why care about having a basis?

If $\{\vec{v}_1, \ldots, \vec{v}_r\}$ is a basis for the subspace V, then any vector $\vec{v}$ in V can be written <u>uniquely</u> as a linear combination

$$\vec{v} = c_1\vec{v}_1 + \cdots + c_r\vec{v}_r$$