

WORKSHEET 3/1/23
MATH 2331, SPRING 2023

(1) Find the inverse of the matrix $\frac{1}{5} \begin{bmatrix} 3 & -4 \\ 4 & 3 \end{bmatrix}$.

(2) Find the inverse of the matrix $\begin{bmatrix} \cos \theta & -\sin \theta \\ \sin \theta & \cos \theta \end{bmatrix}$.

(3) Based on your answers to #1 and #2, try to guess the inverses of these matrices:

$$\frac{1}{2} \begin{bmatrix} 1 & 1 & 1 & 1 \\ 1 & -1 & -1 & 1 \\ 1 & 1 & -1 & -1 \\ 1 & -1 & 1 & -1 \end{bmatrix} \quad \begin{bmatrix} 0 & 1 & 0 \\ 0 & 0 & 1 \\ 1 & 0 & 0 \end{bmatrix}.$$

(4) Calculate $\vec{v}^T \vec{w}$, where $\vec{v} = \begin{bmatrix} 1 \\ 2 \\ 3 \end{bmatrix}$ and $\vec{w} = \begin{bmatrix} 4 \\ 5 \\ 6 \end{bmatrix}$. What do you notice?

(5) Calculate $Q^T Q$, where $Q = \begin{bmatrix} 1 & 0 \\ 0 & 1 \\ 0 & 0 \end{bmatrix}$.

(6) Based on your answer to #5, guess a formula for $Q^T Q$ when the columns of Q are orthonormal.

(7) Based on your answer to #6, is Q invertible?