Last time

- Informal discussion of systems of linear equations
- Toy examples
- First steps toward solving

A __linear equation__ is one of the form

$$a_1x_1 + a_2x_2 + \cdots + a_nx_n = b,$$

where $a_1, \ldots, a_n$ ("coefficients") and b are (real) numbers and $x_1, \ldots, x_n$ are variables. A __system__ of linear equations is a collection of linear equations in the same variables.

<u>Ex</u> $5P_1 - P_2 = 84$

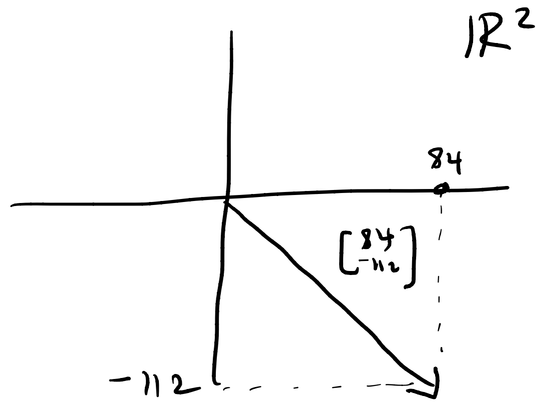
$$P_1 - 3P_2 = -112$$

Usually the variables are understood, so we record the whole system as an array, called a matrix, in one of two ways:

$$\begin{bmatrix} 5 & -1 \\ 1 & -3 \end{bmatrix} + \begin{bmatrix} 84 \\ -112 \end{bmatrix} \quad \text{OR} \quad \left[\begin{array}{cc|c} 5 & -1 & 84 \\ 1 & -3 & -112 \end{array}\right]$$

"coefficient matrix" a "column vector" "augmented matrix"

Column vectors are also represented visually as arrows:



From this standpoint, a linear equation represents a line (or plane, or...) and a system represents their intersection (more later).

Last time, we saw how to solve the system

$$5P_1 - P_2 = 84$$
$$P_1 - 3P_2 = -112$$

by elimination of variables. We used three "moves"

1. swap two equations
2. add a multiple of an equation to another equation
3. scale an equation

In terms of matrices, these moves look like <u>row operations</u>:

(I) <u>swap</u> two rows

(II) <u>add</u> a multiple of a row to another

(III) <u>scale</u> a row

$$\left[\begin{array}{cc|c} 5 & -1 & 84 \\ 1 & -3 & -112 \end{array}\right] \overset{\text{I}}{\rightsquigarrow} \left[\begin{array}{cc|c} 1 & -3 & -112 \\ 5 & -1 & 84 \end{array}\right]$$

$$\overset{\text{II}}{\rightsquigarrow} \left[\begin{array}{cc|c} 1 & -3 & -112 \\ 0 & 14 & 644 \end{array}\right] \overset{\text{III}}{\rightsquigarrow} \left[\begin{array}{cc|c} 1 & -3 & -112 \\ 0 & 1 & 46 \end{array}\right] \overset{\text{II}}{\rightsquigarrow} \left[\begin{array}{cc|c} 1 & 0 & 26 \\ 0 & 1 & 46 \end{array}\right]$$

$$\Rightarrow \begin{bmatrix} P_1 \\ P_2 \end{bmatrix} = \begin{bmatrix} 26 \\ 46 \end{bmatrix}.$$

We use row operations to transform a matrix into one that represents an "easy-to-solve" system.

Def A matrix is in reduced row-echelon form if it satisfies the following conditions.

(A) The first nonzero entry in each row (if there is one) is 1 ("pivot", "leading 1")

(B) If a column has a pivot, its other entries are 0.

(C) If a row has a pivot, all previous

rows have pivots farther to the left.

Fact Every matrix A has a unique RREF, which can be obtained from A by applying row operations.

$$\begin{bmatrix} 1 & ? & 0 & 0 & ? \\ & & 1 & 0 & ? \\ & 0 & & 1 & \end{bmatrix}$$

This procedure is called "row reduction" or "Gauss-Jordan elimination".