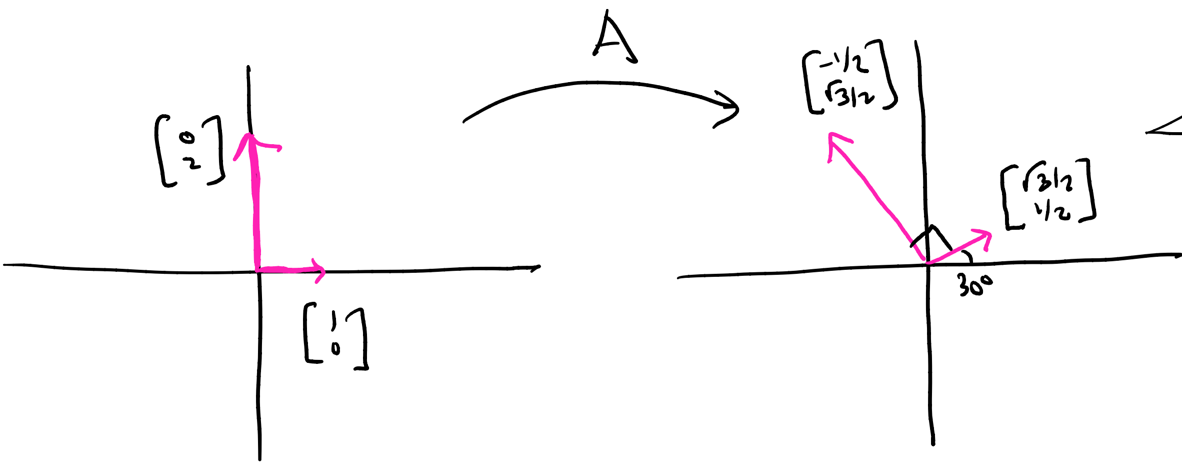
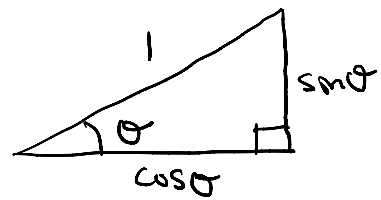
Last time

- Matrix multiplication
- Distribution vectors + transition matrices
- Geometry of linear transformations

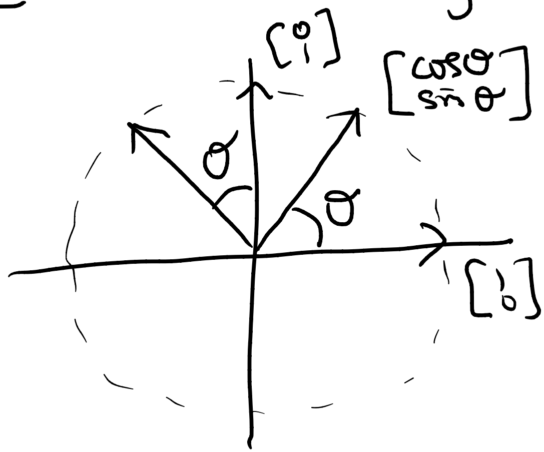
Ex $A = \begin{bmatrix} \sqrt{3}/2 & -1/2 \\ 1/2 & \sqrt{3}/2 \end{bmatrix}$

$$\sqrt{3}/2 = \cos(30^\circ) = \cos(\pi/6)$$
$$1/2 = \sin(30^\circ) = \sin(\pi/6)$$





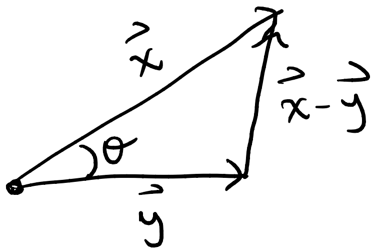
Ex Rotation by an angle θ



This transformation sends $\begin{bmatrix}1\\0\end{bmatrix}$ to the vector on the unit circle at angle θ, which is $\begin{bmatrix}\cos\theta\\\sin\theta\end{bmatrix}$. Similarly, it sends $\begin{bmatrix}0\\1\end{bmatrix}$ to the unit vector at angle $\theta+\pi/2$, which is $\begin{bmatrix}\cos(\theta+\pi/2)\\\sin(\theta+\pi/2)\end{bmatrix}=\begin{bmatrix}-\sin\theta\\\cos\theta\end{bmatrix}$.

$$\boxed{A=\begin{bmatrix}\cos\theta & -\sin\theta\\ \sin\theta & \cos\theta\end{bmatrix}}$$

Aside (dot products and cosines)



$$|\vec{x}-\vec{y}|^2 = |\vec{x}|^2 + |\vec{y}|^2 - 2|\vec{x}||\vec{y}|\cos\theta$$ (Law of cosines)

$$|\vec{x}|^2 = \vec{x}\cdot\vec{x} \text{ etc.}$$

$$\Longrightarrow |\vec{x}|^2+|\vec{y}|^2 - 2|\vec{x}||\vec{y}|\cos\theta = |\vec{x}-\vec{y}|^2$$

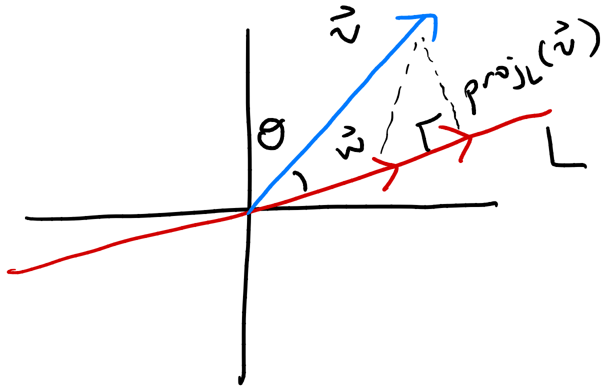
$$= (\vec{x}-\vec{y})\cdot(\vec{x}-\vec{y})$$

$$= \vec{x}\cdot\vec{x} + \vec{y}\cdot\vec{y} - 2\vec{x}\cdot\vec{y}$$

$$= |\vec{x}|^2 + |\vec{y}|^2 - 2\vec{x}\cdot\vec{y}$$

$$\boxed{\cos\theta = \frac{\vec{x}\cdot\vec{y}}{|\vec{x}||\vec{y}|}}$$

Ex orthogonal projection onto a line L



If $\vec{w}$ is a nonzero vector parallel to L,

$$\text{proj}_L(\vec{v}) = \frac{|\text{proj}_L(\vec{v})|}{|\vec{w}|}\vec{w}$$

$$\cos\theta = \frac{|\text{proj}_L(\vec{v})|}{|\vec{v}|}$$

So $|\text{proj}_L(\vec{v})| = |\vec{v}|\cos\theta$

$$= |\vec{v}|\frac{\vec{v}\cdot\vec{w}}{|\vec{v}||\vec{w}|} \implies \boxed{\text{proj}_L(\vec{v}) = \frac{\vec{v}\cdot\vec{w}}{|w|^2}\vec{w}}$$