WORKSHEET 9/27/23
MATH 2331, FALL 2023

(1) Decide whether each matrix is invertible. If it is, find the inverse matrix. Don't work hard if you don't have to!

(a) $\begin{bmatrix} 1 & 1 & 1 \\ 0 & 0 & 0 \\ 3 & 8 & 2 \end{bmatrix}$

(b) $\begin{bmatrix} 1 & 0 & 1 \\ 2 & 0 & 2 \\ 3 & 0 & 2 \end{bmatrix}$

(c) $\begin{bmatrix} 1 & 1 & 1 \\ 2 & 3 & 2 \end{bmatrix}$

(d) $\begin{bmatrix} 1 & 1 & 1 \\ 2 & 3 & 2 \\ 3 & 8 & 2 \end{bmatrix}$

(2) Let T be a linear transformation, $\vec{v}$ a vector in $\operatorname{im}(T)$ and $\vec{w}$ a vector in $\ker(T)$. How many entries do $\vec{v}$ and $\vec{w}$ have?

(3) Let A be an $m \times n$ matrix.
   (a) Suppose that $\ker(A) = \{\vec{0}\}$. What can you say about the rank of A?
   (b) Suppose that $\operatorname{im}(A) = \mathbb{R}^m$. What can you say about the rank of A?

(4) Think of an $m \times n$ matrix A with $\operatorname{im}(A) = \mathbb{R}^m$ and $\ker(A) \neq \{0\}$.

(5) Think of an $m \times n$ matrix B with $\ker(A) = \{0\}$ and $\operatorname{im}(A) \neq \mathbb{R}^m$.