WORKSHEET 9/28/23
MATH 2331, FALL 2023

(1) Let A be an $m \times n$ matrix.
   (a) Suppose that $\ker(A) = \{\vec{0}\}$. What can you say about the rank of A?
   (b) Suppose that $\text{im}(A) = \mathbb{R}^m$. What can you say about the rank of A?

(2) Think of an $m \times n$ matrix A with $\text{im}(A) = \mathbb{R}^m$ and $\ker(A) \neq \{0\}$.

(3) Think of an $m \times n$ matrix B with $\ker(A) = \{0\}$ and $\text{im}(A) \neq \mathbb{R}^m$.

(4) Describe the span of $\left\{ \begin{bmatrix} 1 \\ 0 \end{bmatrix}, \begin{bmatrix} 0 \\ 1 \end{bmatrix} \right\}$.

(5) Describe the span of $\left\{ \begin{bmatrix} 1 \\ 0 \end{bmatrix}, \begin{bmatrix} 2 \\ 0 \end{bmatrix} \right\}$.

(6) Find a collection of vectors that spans the image of each matrix. Be as efficient as you can!

   (a) $\begin{bmatrix} 1 & 2 & 3 & 4 & 5 & 6 & 7 & 8 & 9 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \end{bmatrix}$

   (b) $\begin{bmatrix} 1 & 0 & 1 \\ 0 & 1 & 1 \end{bmatrix}$

   (c) $\begin{bmatrix} 1 & 2 & 3 \\ 0 & 1 & 2 \\ 0 & 0 & 1 \end{bmatrix}$

(7) Are the the column vectors of the matrix from part (c) linearly independent?