WORKSHEET 10/2/23 MATH 2331, FALL 2023

You may take for granted that the reduced row-echelon form of the matrix $\begin{bmatrix} 1 & 4 & 7 \\ 2 & 5 & 8 \\ 3 & 6 & 9 \end{bmatrix}$ is the

 $matrix \begin{bmatrix} 1 & 0 & -1 \\ 0 & 1 & 2 \\ 0 & 0 & 0 \end{bmatrix}.$

- (1) Are the vectors $\vec{v}_1 = (1, 2, 3)$, $\vec{v}_2 = (4, 5, 6)$, and $\vec{v}_3 = (7, 8, 9)$ linearly independent?
- (2) How large could a collection of linearly independent vectors in \mathbb{R}^m be?
- (3) Do the vectors $\vec{v}_1 = (1, 2, 3)$, $\vec{v}_2 = (4, 5, 6)$, and $\vec{v}_3 = (7, 8, 9)$ span \mathbb{R}^3 ?
- (4) How small could a set of vectors spanning \mathbb{R}^m be?
- (5) Suppose that $\vec{w_1}$, $\vec{w_2}$, and $\vec{w_3}$ are linearly independent vectors. How many ways could there be to write another vector \vec{w} as a linear combination of these vectors?

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