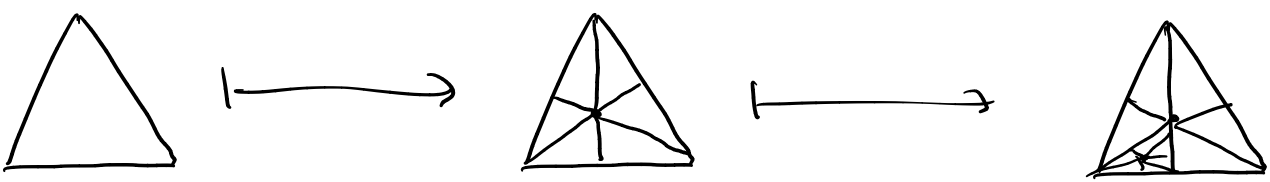
Last time

- Generalized Jordan curve & applications
- H_* and open exhaustions

Subdivision thm If $X = \bigcup_{\alpha \in A} \mathring{U}_\alpha$, then the inclusion $C_*^{\mathcal{U}}(X) \subseteq C_*(X)$ is a chain homotopy equivalence.

Idea) Barycentric subdivision



__Idea 2__ Given $\sigma: \Delta^n \to X$, $\sigma = \sigma_\#(\mathrm{id}_{\Delta^n})$, so we may work with special simplices on Δ^n.

__Def__ Fix $K \subseteq \mathbb{R}^N$ convex. Given $v_0, \dots, v_m \in K$, the __affine m-simplex__ $[v_0, \dots, v_m]$ is the map

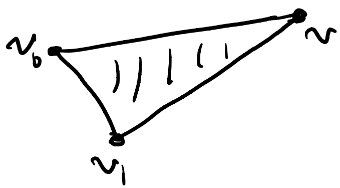
$$\begin{aligned} \Delta^m &\longrightarrow K \\ (t_0, \dots, t_m) &\longmapsto \sum_{i=0}^m t_i v_i \end{aligned}$$

__Ex__ $\mathrm{id}_{\Delta^n} = [e_0, \dots, e_n]$

Write $L_*(K) \subseteq C_*(K)$ for the subcomplex spanned by the affine simplices.

Def Given $\sigma=[v_0,\dots,v_m]$ and $v\in K$, the <u>cone</u> on σ from v is the affine $(m+1)$-simplex

$$v\sigma=[v,v_0,\dots,v_m].$$



Lemma 1

$$\partial v\sigma = \begin{cases} \sigma - v(\partial\sigma) & m>0 \\ \sigma - \varepsilon(\sigma)[v] & m=0, \end{cases}$$

where $\varepsilon(\sum n_i[v_i])=\sum n_i$.

Proof Exercise. $\square$

Def The <u>barycenter</u> of $\sigma=[v_0,\dots,v_m]$ is

$$\underline{\sigma}=\sum_i \frac{1}{m+1}v_i\in K.$$

The <u>barycentric subdivision</u> of σ is the chain $S\sigma \in L_m(K)$ defined inductively by

$$S\sigma = \begin{cases} \sigma & m=0 \\ \underline{\sigma}\, S(\partial\sigma) & m>0 \end{cases}.$$

<u>Lemma 2</u> $S : L_*(K) \to L_*(K)$ is a chain map.

<u>Proof</u> Exercise using Lemma 1. $\square$

<u>Prop</u> S is chain homotopic to the identity.

<u>Proof</u> Define $T : L_m(K) \to L_{m+1}(K)$ inductively by

$$T\sigma = \begin{cases} 0 & m=0 \\ \underline{\sigma}\,(S\sigma - \sigma - T\partial\sigma) & m>0 \end{cases}$$

and check that $\partial T + \partial T = S - \mathrm{id}$ (exercise).

$\square$